## Vapour power cycle: Superheat, reheat, 3-stage regenerative feed water preheating

(This is an updated version of Dr. Sauerhering's (LTT ISuT) original solution)

## Question

A boiler generates a superheated steam flow with 2100 t/hour at 600  $^{\circ}$ C and 200 bar. The steam is expanded in a turbine, with three tappings for bleed steam for three regenerative feed water preheating stages. In addition, the steam has to be reheated after the first expansion stage to 600  $^{\circ}$ C. The condensation pressure is 0.05 bar.

a) Draw a schematic of the plant and draft a T-s diagram

b) Calculate the thermal efficiency. Assume isentropic expansions in the turbine, and neglect the pump work

c) Calculate the turbine power output

Use the T - s chart to determine the thermal properties of the steam and feed water. Calculate the intermediate pressures for the regenerative feed water preheating using the following relation for the enthalpy difference:

$$\Delta h' = \frac{h'_{max} - h'_{min}}{n+1} \tag{1}$$

where  $h'_{max}$  and  $h'_{min}$  are respectively the maximum and minimum enthalpies of water at the maximum and minimum pressures in the cycle, and n is the number of feed water preheating steps.

## Worked solution

Schematic (Fig. 1): after the first expansion stage,  $y_3$  kg of steam is put back in the superheater and reheated to 600 °C.  $y - y_3$  kg of steam is taken to the feed water heater labelled A. At each subsequent stage, a part of the steam



Figure 1: Plant schematic with thermal states and mass fractions y

 $y_n$  is expanded in the next turbine, and the bleed steam is taken to the next upstream feed water heater in the feed water line.

First, the thermal state at  $P_1 = P_{max} = 20$  Mpa (200 bar),  $T_1 = 600$  °C can be found from the T - s chart (see P. 6 of this pdf):

 $h_1 = 3550 \text{ kJ/kg}, s_1 = 6.5 \text{ kJ/kgK}$ 

To calculate the pressures for the feed water preheating, we need  $h'_{max}$  and  $h'_{min}$ . These are respectively the maximum and minimum enthalpies of water at the maximum and minimum pressures in the cycle. From the T-s chart on the saturated liquid line:

 $h'_{max} = 1800 \text{ kJ/kg at } P_{max} = 20 \text{ Mpa},$ 

 $h'_{min} = 150 \text{ kJ/kg}$  at  $P_{min} = 0.005 \text{ Mpa} (0.05 \text{ bar})$ .

There are n=3 feed water preheating steps so from Eq. 1:

 $\Delta h' = 413 \text{ kJ/kg}.$ 

This can be used to calculate the thermal state in the feed water line after each of the preheaters.  $h'_{min} = h_7 = h_8 = 150 \text{ kJ/kg}$  (neglecting the pumping work between 7 and 8. NOTE that subsequently in the schematic the difference in the enthalpies before and after the pumps at points 9, 10 and 11 in the feed water line have been ignored i.e. single thermal states at each point 9, 10, 11 are assumed. The nomenclature is simplified accordingly). Therefore:

 $h_9 = h_8 + \Delta h' = 563 \text{ kJ/kg}$ . From the T - s chart,  $P_9 = 0.3 \text{ Mpa}$ .

In the same way for the other points in the feed water line:

 $h_{10}=h_9+\Delta h'=976 \text{ kJ/kg}; P_{10}=3.0 \text{ Mpa}$ 

 $h_{11}=h_{10}+\Delta h'=1389 \text{ kJ/kg}; P_{11}=10 \text{ Mpa}$ 

After the first expansion,  $s_1 = s_2=6.5 \text{ kJ/kgK}$  (assuming isentropic expansion) and  $P_2 = P_{11}=10$  Mpa. From the T - s chart:

 $h_2 = 3325 \text{ kJ/kg}.$ 

Then after superheating  $y_3$  kg steam back to  $T_3=600$  °C at constant pressure  $(P_2 = P_3=10 \text{ Mpa})$ , from the T-s chart:

$$h_3 = 3625 \text{ kJ/kg}; s_3 = 6.9 \text{ kJ/kgK}.$$

Isentropic expansions are assumed  $(s_3 = s_4 = s_5 = s_6)$  and so the enthalpies for states 4, 5 and 6 can be found from the T-s chart at the pressures where the steam is bled off:

 $P_4 = P_{10} = 3.0 \text{ Mpa}; h_4 = 3175 \text{ kJ/kg}$ 

 $P_5 = P_9 = 0.3 \text{ Mpa}; h_5 = 2600 \text{ kJ/kg}$ 

 $P_6 = P_7 = P_{min} = 0.005 \text{ Mpa}; h_6 = 2100 \text{ kJ/kg} (dryness fraction x_6 = 0.81).$ 

All the steam is expanded through the first turbine, but this is not so for the other three turbines. Therefore to calculate the work outputs and heat addition(s) and rejection, the mass fractions  $y_n$  must be known. These are found by balancing the enthalpies in each feed water heater as shown in Fig. 2. Starting with the feed water heater labelled A in the schematic:

 $yh_{11} = (y-y_3)h_2 + y_3h_{10} \implies h_{11} = (1-y_3)h_2 + y_3h_{10}$ , per unit mass y=1 kg of steam.



Figure 2: Enthalpy balance in a feedwater preheater

Rearranging for  $y_3$ :

$$y_3 = \frac{h_{11} - h_2}{h_{10} - h_2} = 0.820 \tag{2}$$

This means  $y - y_3 = 18\%$  by mass of steam is bled off after the first expansion and  $y_3 = 82\%$  re-enters the superheater and the next turbine stage.

By balancing the enthalpies in the same way for each preheater, the mass fractions  $y_2$  and  $y_1$  can be found, e.g. for preheater labelled B in the schematic,  $y_3h_{10} = (y_3 - y_2)h_4 + y_2h_9$  and so on, to yield the answers:

 $y_2 = 0.687; y_1 = 0.569$ 

Finally with the enthalpies of the working fluid in every state, together with the mass fractions of steam at each stage, the work outputs and heat addition(s) and rejection can be determined. All the steam is expanded through the first turbine:

 $W_{1-2} = h_2 - h_1 = -225 \text{ kJ/kg}$ 

However only  $y_3$  of steam is expanded through the second turbine:

 $W_{3-4} = (h_4 - h_3)y_3 = -369 \text{ kJ/kg}$ 

and in the same fashion for the two downstream turbines, to find:

 $W_{4-5}$ =-395 kJ/kg;  $W_{5-6}$ =-285 kJ/kg;  $\Sigma W$ =-1260 kJ/kg (neglecting the pumping work).

The heat addition in the boiler and superheater is supplied to all the working

fluid:

 $Q_{11-1} = h_1 - h_{11} = 2161 \text{ kJ/kg}$ 

Only  $y_3$  of the steam is reheated following the first expansion:

 $Q_{2-3}{=}~245~{\rm kJ/kg};\,Q_{in}{=}~2407~{\rm kJ/kg}$ 

and the heat is rejected from  $y_1$  of fluid in the condenser:

$$Q_{6-7} = -1110 \text{ kJ/kg}$$

The efficiency is:

$$\eta = \frac{|\sum W|}{Q_{in}} = 52.3\% \tag{3}$$

The power output, for a mass flow 2100 tons/hr = 583 kg/s, is 735 MW.

